Mathematics:

1. $MC = Q^2 - 50Q + 700 \quad \Rightarrow \quad TC(Q) = \int MC(Q)\,\mathrm{d}Q = \frac{1}{3}Q^3 - 25Q^2 + 700Q + C$

$TC(0) = 2000 \quad \Rightarrow \quad C = 2000 \quad \Rightarrow \quad TC(Q) = \frac{1}{3}Q^3 - 25Q^2 + 700Q + 2000$

$TC(15) = 8000$

False

2. $MC'(Q) = 2Q - 50 \qquad MC''(Q) = 2$

$MC'(Q) = 0 \quad \Rightarrow \quad 2Q - 50 = 0 \quad \Rightarrow \quad Q = 25$

$MC''(Q) > 0 \quad \Rightarrow$ Minimum at $Q = 25$.

$MC(25) = 75$

True

3. Demand function $P = 1700 - 40Q$

Total revenue: $TR = P \cdot Q = 1700Q - 40Q^2$

Marginal revenue: $MR = TR'(Q) = 1700 - 80Q$

Profit is maximized when $MR = MC \quad \Rightarrow \quad 1700 - 80Q = Q^2 - 50Q + 700 \quad \Rightarrow$

$Q^2 + 30Q - 1000 = 0 \quad \Rightarrow \quad (Q+50)(Q-20) = 0 \quad \Rightarrow \quad Q = 20$

$P = 1700 - 40 \cdot 20 = 1700 - 800 = 900$

True

4. $y(x) = xe^{4x} \quad \Rightarrow \quad y'(x) = 1 \cdot e^{4x} + x \cdot 4e^{4x} = e^{4x} + 4xe^{4x}$

$y''(x) = 4e^{4x} + 4e^{4x} + 4x \cdot 4e^{4x} = 8e^{4x} + 16xe^{4x}$

$y''(x) = 0 \quad \Rightarrow \quad 8e^{4x} + 16xe^{4x} = 0$

The x-coordinate of a point of inflection is a solution of $y''(x) = 0 \quad \Rightarrow$

$8e^{4x} + 16xe^{4x} = 0 \quad \Rightarrow \quad 8e^{4x}\left(1 + 2x\right) = 0 \quad \Rightarrow \quad 1 + 2x = 0 \quad \Rightarrow \quad x = -\frac{1}{2}$

True

5. The correct statement is: If $y(x)$ attains a local maximum at $x = a$, then $y'(a) = 0$ and $y''(a) < 0$.

False

6. $\int \frac{10}{2x+1}\mathrm{d}x = \int 10 \cdot \frac{1}{2x+1}\mathrm{d}x = 10 \cdot \frac{1}{2}\ln|2x+1| + c = 5\ln|2x+1| + c$

False

7. If $I(t)$ is the total value of the investment after t years, then $I(t) = 50\,000e^{0.06t}$

$I(10) = 50\,000e^{0.6} \approx 91105.94$

False

8. $APR = \left(1 + \frac{0.06}{4}\right)^4 - 1 \approx 0.0614$. So the annual percentage rate is 6.14%.

True

Applied Business Methods:

1a

An independent samples design. The sample members are selected independently and not in pairs.

1b

$H_0 : \mu_1 = \mu_2$, $H_1 : \mu_1 \neq \mu_2$.

Test statistic $T_p = \frac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2)$,

where $S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = 64.81$.

Under the null hypothesis T_p has a $t_{n_1+n_2-2} = t_{198}$ distribution. Intuitive rejection region area: reject H_0 when $t_{obs} >> 0$ or $t_{obs} << 0$ Significance level = 5%

Critical values: $t_{obs} > 1.972$ or $t_{obs} < -1.972$

$T_p = \frac{23.71 - 23.61 - 0}{8.0505\sqrt{\frac{1}{100} + \frac{1}{100}}} = 0.088$ Since 0.088 is between -1.972 and +1.972, there is not enough evidence to infer that the population means differ at the 5% significance level.

1c

$H_0 : \sigma_1^2 / \sigma_2^2 = 1$ $H_1 : \sigma_1^2 / \sigma_2^2 \neq 1$

Test statistic $F_{obs} = S_1^2 / S_2^2$.

Under the null hypothesis F_{obs} has a $F_{100-1,100-1} = F_{99,99}$ distribution. Intuitive rejection region area: reject H_0 when $F_{obs} >> 1$ or $F_{obs} << 1$ Significance level = 5%

Critical values: $F > F_{\alpha/2,\nu_1,\nu_2} = F_{.025,99,99} = 1.49$ (interpolation between 1.43 and 1.58 should give a similar result) or

$F < F_{1-\alpha/2,\nu_1,\nu_2} = 1/F_{\alpha/2,\nu_2,\nu_1} = 1/F_{.025,99,99} = 1/1.49 = .67$

$F_{obs} = s_1^2 / s_2^2 = 5.5017^2 / 9.9675^2 = 0.30$. Since .30 is smaller than .67,

there is enough evidence to infer that the population variances differ at the 5% significance level.s