1. Uitwerkingen

1. $\frac{dx}{dt} = 4x$

   Substitutie van $x(t) = 4e^t$ in de gegeven differentiaalvergelijking geeft

   $$\frac{d\left(4e^t\right)}{dt} = 4 \cdot 4e^t \quad \Rightarrow \quad 4e^t = 16e^t$$

   Onjuist.

2. $\frac{dx}{dt} = \frac{4t}{t^2+1} \cdot x$

   Substitutie van $x(t) = 2\ln\left(t^2+1\right)$ in de gegeven differentiaalvergelijking geeft

   $$\frac{d\left(2\ln\left(t^2+1\right)\right)}{dt} = \frac{4t}{t^2+1} \cdot 2\ln\left(t^2+1\right) \quad \Rightarrow \quad \frac{2}{t^2+1} \cdot 2t = \frac{8t\ln\left(t^2+1\right)}{t^2+1} \quad \Rightarrow$$

   $$\frac{4t}{t^2+1} = \frac{8t\ln\left(t^2+1\right)}{t^2+1}$$

   Onjuist.

3. $\frac{dx}{dt} = \frac{4t}{t^2+1} \cdot x \quad \Rightarrow \quad \frac{1}{x} \cdot \frac{dx}{dt} = \frac{4t}{t^2+1} \quad \Rightarrow \quad \int \frac{1}{x} \cdot \frac{dx}{dt}\,dt = \int \frac{4t}{t^2+1}\,dt \quad \Rightarrow$

   $$\int \frac{1}{x}\,dx = \int \frac{4t}{t^2+1}\,dt \quad \Rightarrow \quad \ln|x| = \int \frac{4t}{t^2+1}\,dt$$

   Berekening van $\int \frac{4t}{t^2+1}\,dt$:

   $$\int \frac{4t}{t^2+1}\,dt = \int 2 \cdot \frac{1}{t^2+1} \cdot 2t\,dt$$

   Stel $u = t^2+1 \quad \Rightarrow \quad \frac{du}{dt} = 2t \quad \Rightarrow$

   $$\int 2 \cdot \frac{1}{t^2+1} \cdot 2t\,dt = \int 2 \cdot \frac{1}{u} \cdot \frac{du}{dt}\,dt = \int 2 \cdot \frac{1}{u}\,du = 2\ln|u| = 2\ln\left(t^2+1\right)$$

$$\ln|x| = \int \frac{4t}{t^2+1}\,dt \quad \Rightarrow \quad \ln|x| = 2\ln\left(t^2+1\right)+c \quad \Rightarrow \quad \ln|x| = \ln\left(t^2+1\right)^2 + c \quad \Rightarrow$$

$$|x| = e^{\ln\left(t^2+1\right)^2+c} \quad \Rightarrow \quad |x| = e^c \cdot e^{\ln\left(t^2+1\right)^2} \quad \Rightarrow \quad |x| = e^c\left(t^2+1\right)^2 \quad \Rightarrow$$

$$x(t) = \pm e^c \cdot \left(t^2+1\right)^2 \quad \Rightarrow \quad x(t) = C\left(t^2+1\right)^2$$

$$x(0) = 5 \quad \Rightarrow \quad C = 5 \quad \Rightarrow \quad x(t) = 5\left(t^2+1\right)^2$$

$$x(3) = 5\cdot 10^2 = 500$$

Juist.

2. Uitwerking

$$f(x,y) = 3x^2+7xy+3y^2+16 \qquad \phi(x,y) = x^2+y^2-200$$

1. Vergelijkingen van Lagrange:

$$\begin{cases} \dfrac{\partial f}{\partial x} - \lambda\dfrac{\partial \phi}{\partial x} = 0 \\ \dfrac{\partial f}{\partial y} - \lambda\dfrac{\partial \phi}{\partial y} = 0 \\ \phi(x,y) = 0 \end{cases} \quad \Rightarrow \quad \begin{cases} 6x+7y-2x\lambda = 0 \\ 7x+6y-2y\lambda = 0 \\ x^2+y^2-200 = 0 \end{cases}$$

Onjuist.

2. $$\begin{cases} 7x^2-7y^2 = 0 \\ x^2+y^2 = 200 \end{cases}$$

Uit $7x^2-7y^2=0$ volgt $x^2=y^2$.

Substitutie in $x^2+y^2=200$ geeft

$$2x^2 = 200 \quad \Rightarrow \quad x^2 = 100 \quad \Rightarrow \quad x=10 \quad \text{of} \quad x=-10.$$

$$x = 10 \quad \Rightarrow \quad y^2 = 100 \quad \Rightarrow \quad y = 10 \quad \text{of} \quad y=-10$$

$$x = 10 \quad \Rightarrow \quad y^2 = 100 \quad \Rightarrow \quad y = 10 \quad \text{of} \quad y=-10$$

Hieruit volgt: Er zijn vier stationaire punten. Uit het verloop van de functiewaarden van $f(x, y)$ over de beperkende voorwaarde volgt: de functie $f(x, y)$ heeft onder de beperkende voorwaarde $\phi(x, y) = 0$ in precies vier punten een extreme waarde.

Juist.

3. $f(10,10) = 300 + 700 + 300 + 16 = 1316$

$f(-10,10) = 300 - 700 + 300 + 16 = -84$

$f(-10,-10) = 300 + 700 + 300 + 16 = 1316$

$f(10,-10) = 300 - 700 + 300 + 16 = -84$

Juist.

3. Uitwerking

a) $$\begin{array}{c} \\ G_1 \\ G_2 \end{array}\begin{array}{c} H_1\ H_2\ H_3 \\ \begin{pmatrix} 4 & 3 & 1 \\ 3 & 3 & 4 \end{pmatrix} \end{array} \cdot \begin{array}{c} \\ H_1 \\ H_2 \\ H_3 \end{array}\begin{array}{c} E_1\ \ E_2 \\ \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 1 & 1 \end{pmatrix} \end{array} = \begin{array}{c} \\ G_1 \\ G_2 \end{array}\begin{array}{c} E_1\ \ E_2 \\ \begin{pmatrix} 12 & 14 \\ 13 & 16 \end{pmatrix} \end{array}$$

b) $A = \begin{pmatrix} 12 & 14 \\ 13 & 16 \end{pmatrix}$

$$\left(\begin{array}{cc|cc} 12 & 14 & 1 & 0 \\ 13 & 16 & 0 & 1 \end{array}\right) \underset{\rightarrow}{\text{rij } 1 - \text{rij } 2} \left(\begin{array}{cc|cc} -1 & -2 & 1 & -1 \\ 13 & 16 & 0 & 1 \end{array}\right) \underset{\rightarrow}{-1 \cdot \text{rij } 1}$$

$$\left(\begin{array}{cc|cc} 1 & 2 & -1 & 1 \\ 13 & 16 & 0 & 1 \end{array}\right) \underset{\rightarrow}{\text{rij } 2 - 13 \cdot \text{rij } 1} \left(\begin{array}{cc|cc} 1 & 2 & -1 & 1 \\ 0 & -10 & 13 & -12 \end{array}\right) \underset{\rightarrow}{\text{rij } 2 / -10}$$

$$\left(\begin{array}{cc|cc} 1 & 2 & -1 & 1 \\ 0 & 1 & -1,3 & 1,2 \end{array}\right) \underset{\rightarrow}{\text{rij } 1 - 2 \cdot \text{rij } 2} \left(\begin{array}{cc|cc} 1 & 0 & 1,6 & -1,4 \\ 0 & 1 & -1,3 & 1,2 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 1,6 & -1,4 \\ -1,3 & 1,2 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1,6 & -1,4 \\ -1,3 & 1,2 \end{pmatrix} \cdot \begin{pmatrix} 4860 \\ 5465 \end{pmatrix} = \begin{pmatrix} 125 \\ 240 \end{pmatrix}$$

c) $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1,6 & -1,4 \\ -1,3 & 1,2 \end{pmatrix} \cdot \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = \begin{pmatrix} 1,6g_1 - 1,4g_2 \\ -1,3g_1 + 1,2g_2 \end{pmatrix}$

$$\begin{cases} 1,6g_1 - 1,4g_2 \geq 0 \\ -1,3g_1 + 1,2g_2 \geq 0 \end{cases} \Rightarrow \begin{cases} 1,6g_1 \geq 1,4g_2 \\ 1,2g_2 \geq 1,3g_1 \end{cases} \Rightarrow \begin{cases} \dfrac{16}{14}g_1 \geq g_2 \\ g_2 \geq \dfrac{13}{12}g_1 \end{cases} \Rightarrow$$

$$\begin{cases} \dfrac{16}{14} \geq \dfrac{g_2}{g_1} \\ \dfrac{g_2}{g_1} \geq \dfrac{13}{12} \end{cases} \Rightarrow \quad \frac{13}{12} \leq \frac{g_2}{g_1} \leq \frac{16}{14} \quad \text{ofwel} \quad \frac{91}{84} \leq \frac{g_2}{g_1} \leq \frac{96}{84}$$