Mathematics 1 (E&BE)

1.

$$3^{5x}9^{x} = 3^{5x}3^{2x} = 3^{7x} = 27$$

$$\Rightarrow 7x = \frac{\log(27)}{\log(3)} = 3$$

$$\Rightarrow x = \frac{3}{7}$$

2.

Lokaal extreem punt ($f'(x) = 0$): x = 1

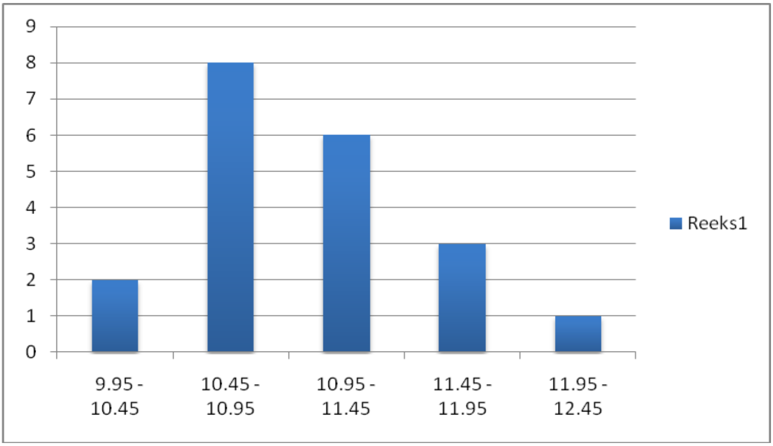
Buigpunten: $f''(x) = 0$: x = -1 of x = 1

3.

$$\int x\sqrt{x}dx = \int x^{1,5}dx = 0,4x^{2,5}$$

Statistiek 1 (BE/FE/A&C)

1.



2.

$$\mu_p = 0{,}20\mu_x + 0{,}80\mu_y = 0{,}20 * 5{,}2\% + 0{,}80 * 13{,}3\% = 11{,}68\%$$

Macro-economie 1 (E&BE/BE/FE/A&C)

1.

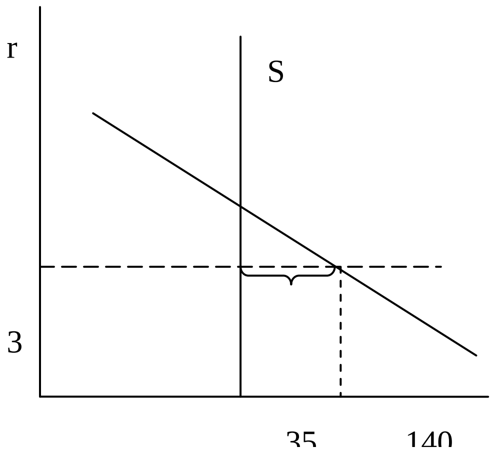
a) $Y_S = Y_D$ or $NX = S - I$

b) The real exchange rate (ε)

c) $NX = 40 - 14.5\varepsilon = S - I$

$40 - 14.5\varepsilon = (Y - C - G) - I = \{300 - 0.8(300 - 50) - 15 - 50\} - (170 - 30) = -105 (\rightarrow$
$\varepsilon = 10)$

d)



e) S-I should equal NX = 0. Investment is determined by the world interest rate so total savings should be equal to 140.

$S_T = S_P + S_G = Y - C - G = Y - 0.8(Y - T) - 15 - G = 0.2Y + 0.8T - 15 - G =$
$60 + 0.8T - 65 = 140 \rightarrow T = 181.25$

So T must rise by 131.25

Wiskunde 2 (BE)

1.

$$|B| = \begin{vmatrix} 4 & 5 & 6 \\ 5 & 6 & 8 \\ 6 & 7 & 9 \end{vmatrix} = 4\begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix} - 5\begin{vmatrix} 5 & 8 \\ 6 & 9 \end{vmatrix} + 6\begin{vmatrix} 5 & 6 \\ 6 & 7 \end{vmatrix} = 1$$

2.

x = 2 and y = -5

3.

Laat $f(x,y) = 3x + 4y$ en $g(x,y) = x^2 + y^2$.

Maximaliseer *f* zodat $g(x,y) = 225$.

- ⇨ max *f* such that $g(x,y) = x^2 + y^2 = 225$
- ⇨ max *f* such that $g(x,y) - 225 = -225 + x^2 + y^2 = 0$

Hence: $L(x,y,\lambda) = 3x + 4y + \lambda(-225 + x^2 + y^2)$

This Lagrangean yields the following conditions:

$$\frac{\partial L}{\partial x} = 3 + 2x\lambda$$

$$\frac{\partial L}{\partial y} = 4 + 2y\lambda$$

$$\frac{\partial L}{\partial \lambda} = -225 + x^2 + y^2$$

From the first two conditions we can derive that:

$$\frac{3}{4} = \frac{2x\lambda}{2y\lambda} = \frac{x}{y}$$

Hence:

$x = \frac{3}{4} y$ and $y = \frac{4}{3} x$

This gives the following solutions:

$$-225 + x^2 + y^2 = 0$$

$$= -225 + x^2 + (\frac{4}{3} x)^2$$

$$= -225 + x^2 + \frac{16}{9} x^2$$

$$= -225 + \frac{25}{9} x^2 = 0$$

$$\Rightarrow \frac{25}{9} x^2 = 225$$

$$\Rightarrow x^2 = 225 * \frac{9}{25}$$

$$\Rightarrow x = \sqrt{225 * \frac{9}{25}} = \sqrt{81} = 9$$

And:

$$-225 + (\frac{3}{4} y)^2 + y^2 = 0$$

$$= -225 + \frac{9}{16} y^2 + y^2$$

$$= -225 + \frac{25}{16} y^2 = 0$$

$$\Rightarrow \frac{25}{16} y^2 = 225$$

$$\Rightarrow y^2 = 225 * \frac{16}{25}$$

$$\Rightarrow y = \sqrt{225 * \frac{16}{25}} = \sqrt{144} = 12$$

Therefore, max f is:

$$3 * 9 + 4 * 12 = 75$$

International Industrial Economics

1.

a) RD_i : $P = 130 - q_1 - q_2$

$TR_1 = 130q_1 - q_1^2 - q_1q_2$

$MR_1 = 130 - 2q_1 - q_2$

$MC_1 = 10$

MR = MC → $q_2 + 2q_1 = 120$ or $q_1 = 60 - ½\ q_2$ and $q_2 = 60 - ½\ q_1$

→ $q_1 = 60 - ½\ (60 - ½\ q_1) = 30 + ¼q_1$ → $¼q_1 = 30$

$q_1 = 40$ and $q_2 = 40$

b) Collusion : determine monopolist output!

$MR = 130 - 2Q$

$MC = 10$

MR = MC → 2Q = 120 Q = 60 → $q_1 = q_2 = 30$

P = 70 $\pi_i = (P - 10)q_i = 60 \times 30 = 1800$

c) Firm 1 cheats: set $q_2 = 30$ (ASSUMPTION)

$RD_1 = 130 - q_1 - q_2 = 100 - q_1$

$MR_1 = 100 - 2q_1$

$MC_1 = 10$

MR = MC → $2q_1 = 90$ $q_1 = 45$

$P = 130 – Q = 130 – (45 + 30) = 55$

Profits: $\pi_1 = (P – 10)q_1 = (55 – 10)45 = 2025$ → gain 225

$\pi_1 = (P – 10)q_2 = (55 – 10)30 = 1350$ → loss 450

Total profits decrease by 225 = 3600 – 3375

d) Long-run : compare net present value

$NPV_{(no\ cheat)} = 1800 + [1 / (1 + r)]\ 1800 + [1 / (1 + r)]^2\ 1800 + ... = 1800 / r$

$NPV_{(cheat\ period\ 0)} = 2025 + [1 / (1 + r)]\ 1600 + [1 / (1 + r)]^2\ 1600 + ... =$

$425 / (1 + r) + 1600 / (1 + r) + [1 / (1 + r)]\ 1600 + [1 / (1 + r)]^2\ 1600 + ... = 425 + 1600 / r$

Cheating is profitable if: $NPV_{(cheat\ period\ 0)} > NPV_{(no\ cheat)}$

$425 / (1 + r) + 1600 / r > 1800 / r$

$425 / (1 + r) > 200 / r$

$425r > 200 (1 + r)$

→ $225r > 200$

→ $r > 200 / 225 = 8 / 9$ = unlikely

2.

a)

Profit firm 1:

$$\pi_1 = \pi_{11} + \pi_{12} = p_1 q_{11} + p_2 q_{12} - c(q)$$

Profit firm 2:

$$\pi_2 = \pi_{21} + \pi_{22} = p_1 q_{21} + p_2 q_{22} - c(q)$$

b)

Market 1:

$$MR_{11} = 150 - 2q_{11} - q_{21} = 30 - \frac{1}{2}q_1 = MC_1$$

$$MR_{21} = 150 - q_{11} - 2q_{21} = 30 - \frac{1}{2}q_2 = MC_2$$

$$\Rightarrow RC_{11} : 120 = 2q_{11} + q_{21} - \frac{1}{2}q_1 = 2q_{11} + q_{21} - \frac{1}{2}(q_{11} + q_{12}) = \frac{3}{2}q_{11} + q_{21} - \frac{1}{2}q_{1}$$

$$\Rightarrow RC_{21} : 120 = \frac{3}{2}q_{21} + q_{11} - \frac{1}{2}q_{22}$$

Market 2:

$$RC_{12} : \frac{3}{2}q_{12} + q_{22} - \frac{1}{2}q_{11} = 120$$

$$RC_{22} : \frac{3}{2}q_{22} + q_{12} - \frac{1}{2}q_{21} = 120$$

As everything is identical, we have

$$120 = \frac{3}{2}q_{11} + q_{21} - \frac{1}{2}q_{12} = \frac{3}{2}q_{11} + q_{11} - \frac{1}{2}q_{11}$$

$$\Rightarrow q_{11} = \frac{120}{2} = 60 = q_{21} = q_{12} = q_{22}$$

c)

$$RC_{11}: 120 = \frac{3}{2}q_{11} + q_{21} - \frac{1}{2}q_{12} = \frac{3}{2}q_{11} + q_{21} - \frac{1}{2}*60 = \frac{3}{2}q_{11} + q_{21} - 30$$

$$\Rightarrow RC_{11}: 150 = \frac{3}{2}q_{11} + q_{21}$$

As cost function etcetera is all identical, we have:

$$RC_{21}: 150 = \frac{3}{2}q_{21} + q_{11}$$

This yields:

$$q_{21} = 150 - \frac{3}{2}q_{11}$$

$$\Rightarrow 150 = \frac{3}{2}(150 - \frac{3}{2}q_{11}) + q_{11}$$

$$\Rightarrow q_{11} = 60 = q_{21}$$

d)

Suppose the quota restricts country 2 firm's sales in country 1 to $q_{21} = 30$.
The new reaction curves can therefore be transformed as follows:

$$RC_{11}: \frac{3}{2}q_{11} + 30 - \frac{1}{2}q_{12} = 120$$

$$\Rightarrow RC_{11} = \frac{3}{2}q_{11} - \frac{1}{2}q_{12} = 90$$

$$RC_{21}: \frac{3}{2}*30 + q_{11} - \frac{1}{2}q_{22} = 120$$

$$\Rightarrow RC_{21}: q_{11} - \frac{1}{2}q_{22} = 75$$

$$RC_{12}: \frac{3}{2}q_{12} + q_{22} - \frac{1}{2}q_{11} = 120$$

$$RC_{22}: \frac{3}{2}q_{22} + q_{12} - \frac{1}{2} * 30 = 120$$

$$\Rightarrow RC_{22}: \frac{3}{2}q_{22} + q_{12} = 135$$

From RC_{22}, we have:

$$q_{12} = 135 - \frac{3}{2}q_{22}$$

And from RC_{21}, we have:

$$q_{11} = 75 + \frac{1}{2}q_{22}$$

Implementing this in RC_{12} yields:

$$RC_{12}: \frac{3}{2}q_{12} + q_{22} - \frac{1}{2}q_{11} = \frac{3}{2}(135 - \frac{3}{2}q_{22}) + q_{22} - \frac{1}{2}(75 + \frac{1}{2}q_{22}) = 120$$

$$= \frac{405}{2} - \frac{9}{4}q_{22} + q_{22} - \frac{75}{2} - \frac{1}{4}q_{22} = 120$$

$$\Rightarrow q_{22} = 45$$

$$\Rightarrow q_{12} = 135 - \frac{3}{2}q_{22} = 135 - \frac{3}{2} * 45 = 67{,}5$$

$$\Rightarrow q_{11} = 75 + \frac{1}{2}q_{22} = 75 + \frac{1}{2} * 45 = 97{,}5$$

And of course $q_{21} = 30$ as implied by the quota imposed.